On the Performance of Cooperative Spectrum Sensing under Quantization

Weijia Han, Jiandong Li, Zan Li, Yan Zhang, and Qin Liu

Abstract

In cognitive radio, the cooperative spectrum sensing (CSS) plays a key role in determining the performance of secondary networks. However, there have not been feasible approaches that can analytically calculate the performance of CSS with regard to the multi-level quantization. In this paper, we not only show the cooperative false alarm probability and cooperative detection probability impacted by quantization, but also formulate them by two closed form expressions. These two expressions enable the calculation of cooperative false alarm probability and cooperative detection probability tractable efficiently, and provide a feasible approach for optimization of sensing performance. Additionally, to facilitate this calculation, we derive Normal approximation for evaluating the sensing performance conveniently. Furthermore, two optimization methods are proposed to achieve the high sensing performance under quantization.

Index Terms

cooperative spectrum sensing, cognitive radio, fusion rule, quantization, optimization.

I. Introduction

Recently, extensive studies found that the scarce spectrum resource licensed to primary users (PUs) are under-utilized significantly [1]–[4]. To alleviate this issue, cognitive radio (CR) technol-

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ogy has been brought as a promising method to make efficient use of the limited radio spectrum. In CR, the secondary users (SUs) are permitted to dynamically utilize the idle frequency spectrum by sensing the reasonable access opportunity. Thus, the accuracy of spectrum sensing plays a key role in determining the performance of CR system. To identify the access opportunity with high correctness, the cooperative spectrum sensing (CSS) has been widely concentrated since it greatly improves the sensing performance.

A. *Related Works*

For CSS, the fusion center makes a final sensing decision following a certain fusion rule, based on the total sensing information reported from each involved cooperative user. Commonly, the CSS with fusion rule are generalized into two major categories: *i)* pre-processing based fusion rule [5]–[13]. The fusion center processes the measurements (or test statistics), which are generated by cooperative users pre-processing the observation, to make final sensing judgment; *ii)* direct-forward based fusion rule [14]–[16]. The fusion center processes the total received samples which are forwarded from each cooperative user to make a final sensing decision. The direct-forward based fusion rule requires a large number of overhead for cooperative users to feed back their collected samples, which means the gain from cooperation can be exhausted by the overhead on data fusion. Hence, when compared with the direct-forward based fusion rule, the pre-processing based fusion rule is more valuable in CR.

As shown in [5], the pre-processing based fusion rule is called as the multi-bit fusion rule based on multi-level quantization. In digital communication, it is not avoidable that the measurements (or test statistics) are quantized and converted into a binary output code for transmission. Hence, the test statistics is transmitted by multiple bits and in quantized manner all the time. On the other hand, both the accuracy and the temporal response are crucial in CSS. The temporal response depicts how fast the primary user can be detected out since the presence of primary user, and it is mainly determined by the quantity of overhead spent on reporting the test statistics. As pointed out in [17], the cooperation overhead makes CSS impractical under large number of cooperative users. Hence, the less overhead on reporting the test statistics achieves the better temporal response. This is the reason why there is an amount of researches concentrating on the CSS with counting rule (or called as decision fusion rule), which is also interpreted as the bi-level quantization based fusion rule [18]. Briefly, the number of

quantization levels is as small as possible from the perspective of reducing the cooperative overhead. However, the limited quantization existing ubiquitously in wireless communication introduces quantization error deteriorating the performance of CSS. Thus, it is necessary to study the impact of quantization on the performance of CSS.

For the CSS with multi-level quantization, current researches have not provide a feasible approach to ensure the false alarm probability and detection probability of CSS can be tractable by closed form expressions [5], [14]. Although [5] showed closed form functions to evaluate the final sensing performance (including cooperative false alarm probability and cooperative detection probability), there are unknown variables in these two functions of which the values can only be obtained by exclusive search. Thus, the computation complexity on evaluating the performance of final decision in [5] is very difficult, causing the related performance evaluation improper online. [14] indicated a two-bit fusion rule based CSS scheme where each cooperative user uses three thresholds to divide the value range of the observed test statistics into four regions with different weights and employs two-bit report to express the region where the value of test statistics falling. However, in [14], the weights are preferred unreasonably and the decision threshold cannot be adjusted. As presented in [14], the fusion rule with multi-level quantization is left for the future work. Since the performance of CSS with quantization error cannot be computed efficiently and presented analytically, the current CSS schemes related to multi-level quantization cannot apply the Neyman-Pearson (NP) criterion and Constant Detection Rate (CDR) criterion reasonably, and the optimization of CSS performance cannot be performed efficiently.

B. Contributions

In CR, the common CSS's application scenario, called as homogeneity scenario in this paper, is that the distances between any SUs are small compared with the distance from any SU to the primary transmitter. Thus, the current literatures [6]–[13] regard the averaged received signal to noise ratio (SNR) at each SU are identical. This paper studies the performance of CSS with multi-level quantization in homogeneity scenario.

The quantization theory is employed to have an insight into the current multi-bit fusion rule. Then the multi-bit fusion rule is interpreted as the multi-level quantization based fusion rule. From the perspective of quantization theory, the study of final sensing performance is to analyze the impact of quantizing the test statistics on the data fusion. Based on this opinion, we derive

the cooperative false alarm probability and cooperative detection probability given by closed form expressions. Especially, this paper does not specify a certain type of detector for the derivation of expressions. Consequently, the derived expressions have the following merits: *i*) they are eligible for arbitrary number of quantization levels; *ii*) they are universal for different types of detector; *iii*) they guarantee both the cooperative false alarm probability and cooperative detection probability can be tractable and optimized reasonably; *iv*) moreover, the derived closed form expressions can be used to analyze the impact of quantization error in different quantization scheme, for example, the uniform quantization scheme.

To facilitate the related performance evaluation, the discrete Fourier transform (DFT) method is introduced to simplify the derived two closed form expression and reduce the corresponding computation complexity. Additionally, the Normal approximations for the obtained two closed form expressions are derived to estimate the CSS sensing performance conveniently. Based on the derived Normal approximations, the optimization of sensing performance are programmed to a linear combination problem which can be solved by semidefinite programs efficiently [16]. Furthermore, when considering the Lloyd [19] quantization scheme, an optimization method is presented by using quasi-convex theory.

C. Paper Structure

The rest of this paper is organized as follows: Section II describes the system model, and formulates the fusion problem in current literatures. Sequentially, Section III presents the performance evaluation of CSS with multi-bit fusion rule. In Section IV, the quantization schemes and related optimization methods are discussed. Followed by conclusion, numerical studies are indicated in Section V.

II. Preliminary and Sensing Model

A. Sensing Model

In CR, SUs are required to sense the presence of distant primary transmitter to ensure the QoS of primary link. Considering a secondary base station and related accessing SUs cooperatively sense the presence of primary user, the CSS problem is modeled by Fig. 1. The sensing objective is to properly determine one of two hypotheses H_0 (denote primary user off) and H_1 (denote primary user on) by analyzing the received signal X_i, where $i \in \{1, 2, \cdots, K\}$ denotes the *i*-th

SU. The K SUs pre-process the observed signal to generate the related test statistics of which the probability density functions (pdfs) are $f(x|H_0)$ and $f(x|H_1)$ for H_0 and H_1 respectively.[1] According to the value of test statistics T_i online, each cooperative SU generates its own report R_i, and then they feed back R_i to secondary base station. After the collection of reports R_i, the secondary base station as the fusion center utilizes the received R_i to make a final decision R_o for the whole cooperative secondary cluster.

In this paper, consider the received signal X_i is independent and identically distributed (i.i.d.) for different cooperative SU, and $\int_{\epsilon}^{+\infty} f(x|H_0)dx < \int_{\epsilon}^{+\infty} f(x|H_1)dx$ for $\forall \epsilon \in (-\infty, +\infty)$. These requirements hold for common detectors employed in CR.

B. Preliminary

For conventional-decision-fusion-rule based CSS, the report R_i of i-th SU is generated as following (1),

$$R_i(T_i) = \begin{cases} 0, \text{ if } T_i < \epsilon_0 \\ 1, \text{ if } T_i \geq \epsilon_0 \end{cases} \tag{1}$$

where ϵ_0 is the sensing threshold, and T_i is the test statistics at the i-th SU. And the fusion center makes final sensing decision according to the number of SU reporting the occurrence of primary user. In fusion center, the final sensing decision strategy $R_o(.)$ is given by

$$R_o(R_i) = \begin{cases} H_0, \text{ if } \sum_{i=1}^{K} R_i < N \\ H_1, \text{ if } \sum_{i=1}^{K} R_i \geq N \end{cases} \tag{2}$$

where N is termed as decision threshold and $N \in \{1, 2, \cdots, K\}$.

According to the probability theory, the related cooperative false alarm and detection probabilities under decision fusion rule are given by

$$P_f = \sum_{k=N}^{K} \binom{K}{k} P(T \geq \epsilon_0|H_0)^k (1 - P(T \geq \epsilon_0|H_0))^{K-k}, \tag{3}$$

$$P_d = \sum_{k=N}^{K} \binom{K}{k} P(T \geq \epsilon_0|H_1)^k (1 - P(T \geq \epsilon_0|H_1))^{K-k} \tag{4}$$

[1]In statistical hypothesis testing, a hypothesis test is typically specified in terms of a test statistic, which is a function of the sample [20].

where P_f and P_d denote the cooperative false alarm probability and cooperative detection probability respectively; T is short for T_i; $P(T \geq \epsilon_0|H_0)$ and $P(T \geq \epsilon_0|H_1)$ are the false alarm probability and detection probability of individual cooperative user respectively.

C. LRT based Multi-bit Fusion Rule

For CSS in homogeneity scenario, [5] presents a certain approach to calculate false alarm probability and detection probability regarding multi-level quantization. Let l denote the number of levels in quantization. The whole range of test statistics is separated into l regions: region 0, region 1, $\cdots$, region $l-1$. In addition, $\{k_0, k_1, \cdots, k_{l-1}\}$ mean the number of SUs whose test statistics falls in region 0, region 1, $\cdots$, region $l-1$ respectively. Thus, according to [5], when the likelihood ratio test (LRT) is used at the fusion center and S represents the set of all $\{k_0, k_1, \cdots, k_{l-1}\}$ satisfying that $\sum_{i=0}^{l-1} k_i = K$ and (5), the final probability of detection and probability of false alarm are given by (6) and (7) respectively.

$$\frac{P(T<\epsilon_0|H_1)^{k_0} P(\epsilon_0 \leq T < \epsilon_1|H_1)^{k_1} \cdots P(\epsilon_{l-2} \leq T|H_1)^{k_{l-1}}}{P(T<\epsilon_0|H_0)^{k_0} P(\epsilon_0 \leq T < \epsilon_1|H_0)^{k_1} \cdots P(\epsilon_{l-2} \leq T|H_0)^{k_{l-1}}} > \frac{C_f P(H_0)}{C_m P(H_1)} \tag{5}$$

where C_f and C_m are weights.

$$\begin{aligned} P_d = & \sum_{\{k_0,k_1,\cdots,k_{l-1}\} \in S} \frac{K!}{\{k_0! k_1! \cdots k_{l-1}!\}} P(T<\epsilon_0|H_1)^{k_0} \\ & \times P(\epsilon_0 \leq T < \epsilon_1|H_1)^{k_1} \cdots P(\epsilon_{l-2} \leq T|H_1)^{k_{l-1}} \end{aligned}, \tag{6}$$

$$\begin{aligned} P_f = & \sum_{\{k_0,k_1,\cdots,k_{l-1}\} \in S} \frac{K!}{\{k_0! k_1! \cdots k_{l-1}!\}} P(T<\epsilon_0|H_0)^{k_0} \\ & \times P(\epsilon_0 \leq T < \epsilon_1|H_0)^{k_1} \cdots P(\epsilon_{l-2} \leq T|H_0)^{k_{l-1}} \end{aligned}. \tag{7}$$

For (6)(7), the set S is obtained by substituting the every permutation of $\{k_0, k_1, \cdots, k_{l-1}\}$ into (5). Evidently, it cannot be efficient to acquire S for inequality (5). Hence, the computation complexity of final detection probability and false alarm probability is high, when regarding the computation of acquiring S. Additionally, the analysis of the sensing performance (P_f, P_d) and the choice of optimal threshold controlling (P_f, P_d) are mathematically intractable in current literatures generally.

III. PERFORMANCE OF CSS

This paper treats the conventional multi-bit fusion rule as the multi-level quantization based fusion rule from the perspective of quantization theory, and then studies the performance of CSS with multi-level quantization mathematically. To facilitate the understanding of our results, the decision fusion rule based CSS is re-interpreted firstly as an instance of using quantization theory, when compared with the conventional interpretation in Section II-B. Sequentially, the cooperative false alarm probability and cooperative detection probability of the multi-level quantization are derived and explained in detail.

A. Interpretation of Decision Fusion Rule from Quantization Theory

In the view of quantization theory, the decision generated in SU can be regarded as a special result of multi-level quantization where $l = 2$, which is called as bi-level quantization. In bi-level quantization scheme, the sensing threshold can also be considered as the quantized threshold to separate the whole value range of test statistics T_i into two non-overlapping regions: Region 0 and Region 1. Correspondingly, there are two quantization levels denoted by q_0 and q_1. The quantization of T_i is formulated by

$$T_i'(T_i) = \begin{cases} q_0, \text{ if } T_i < \epsilon \\ \\ q_1, \text{ if } T_i \geq \epsilon \end{cases} \tag{8}$$

where T_i' is the quantized test statistics in *i*th SU. And the report of individual SU is generated as following

$$R_i(T_i') = \begin{cases} 0, \text{ if } T_i' = q_0 \\ \\ 1, \text{ if } T_i' = q_1 \end{cases} . \tag{9}$$

In the fusion center, the decision fusion can be treated as that the quantized test statistics is combined based on received reports R_i, which is formulated by

$$\mathbb{T} = \sum_{i=1}^{K} T_i' = \sum_{i=1}^{K} q_k|_{k=R_i} = k_0 q_0 + k_1 q_1 \tag{10}$$

where $\mathbb{T}$ is the final test statistics in fusion center; k_0 denotes the number of reports $R_i = 0$, and k_1 denotes the number of reports $R_i = 1$. Then the final decision is made by

$$R_o(\mathbb{T}) = \begin{cases} H_0, \text{ if } \mathbb{T} < N \\ \\ H_1, \text{ if } \mathbb{T} \geq N \end{cases} = \begin{cases} H_0, \text{ if } k_0 q_0 + k_1 q_1 < N \\ \\ H_1, \text{ if } k_0 q_0 + k_1 q_1 \geq N \end{cases} \tag{11}$$

where the final decision threshold N is belong to the set $\{k_0 q_0 + k_1 q_1 : \forall k_0 \in \{0, 1, \cdots, K - 1\}, k_1 = K - k_0\}$. Actually, q_0 and q_1 can be normalized to $\beta_0 = (q_0 - q_0)/(q_1 - q_0) = 0$ and $\beta_1 = (q_1 - q_0)/(q_1 - q_0) = 1$ separately, regardless of their original values. Thus, $\mathbb{T} = k_1 = \sum_1^K R_i$, and we arrive at $\{k_0 q_0 + k_1 q_1 : \forall k_0 \in \{0, 1, \cdots, K - 1\}, k_1 = K - k_0\} := \{1, 2, \cdots, K\}$,

$$R_o = \begin{cases} H_0, \text{ if } \sum_1^K R_i < N \\ \\ H_1, \text{ if } \sum_1^K R_i \geq N \end{cases} . \tag{12}$$

When compared (12) with (2), it is evident to see that, from the perspective of quantization theory, the conventional decision fusion rule is explained correctly.

Based on the quantization theory, the cooperative false alarm probability P_f and cooperative detection probability P_d are expressed by $P(\mathbb{T} \geq N|H_0)$ and $P(\mathbb{T} \geq N|H_1)$ respectively. For $P(\mathbb{T} \geq N|H_0)$, the probability mass function is $P(\mathbb{T} = N|H_0) = P(k_0 q_0 + k_1 q_1 = N|H_0)$. Since $P(k_0 q_0 + k_1 q_1 = N|H_0)$ can be interpreted as the probability of arriving exactly k_1 successes in K trials, we have

$$P(k_0 q_0 + k_1 q_1 = N|H_0) = \frac{K!}{k_1!(K - k_1)!} P(T \geq \epsilon_0|H_0)^{k_1} (1 - P(T \geq \epsilon_0|H_0))^{K-k_1} \quad . \tag{13}$$

Additionally, because of $K = k_0 + k_1$, $k_1 = (N - K q_0)/(q_1 - q_0)$. Then, the corresponding cumulative distribution function is

$$P(k_0 q_0 + k_1 q_1 \geq N|H_y) = \sum_{k_1 = (N - K q_0)/(q_1 - q_0)}^{K} \binom{K}{k_1} P(T \geq \epsilon_0|H_y)^{k_1} (1 - P(T \geq \epsilon_0|H_y))^{K-k_1} \tag{14}$$

where $y \in \{0, 1\}$. Since q_0 and q_1 can be normalized to $\beta_0 = 0$ and $\beta_1 = 1$, the cumulative distribution functions is re-formulated by

$$\begin{aligned} P_Y &= P(k_0 \beta_0 + k_1 \beta_1 \geq N|H_y) = P(k_1 \geq N|H_y) \\ &= \sum_{k_1 = N}^{K} \binom{K}{k_1} P(T \geq \epsilon_0|H_y)^{k_1} (1 - P(T \geq \epsilon_0|H_y))^{K-k_1} \end{aligned} \tag{15}$$

where $Y \in \{d, f\}$, $y \in \{0, 1\}$; $Y = d$ when $y = 1$, and $Y = f$ when $y = 0$. When compared with (3) (4), (15) proves that the conventional decision fusion rule can be interpreted correctly from the perspective of quantization theory.

B. Performance of CSS with Multi-level Quantization

From the perspective of quantization theory, the fusion rule with multi-bits report is interpreted as the multi-level quantization based hypothesis test. In homogeneity scenario, the test statistics T_i is considered to follow a same distribution for different SU. Thus, let T denote the test statistics in every SU. Under multi-level quantization, there are l quantization levels with $l-1$ quantization thresholds. Let the l quantization levels and the $l-1$ quantization thresholds be denoted by $\{q_0, q_1, \cdots, q_{l-1}\}$ and $\{\epsilon_0, \epsilon_1, \cdots, \epsilon_{l-2}\}$ respectively. The $l-1$ quantization thresholds separate the value range of T into l non-overlapping regions. The probability that T falls in a certain region is given as follows,

$$P(\epsilon_{i-1} \leq T < \epsilon_i | H_y) = \int_{\epsilon_{i-1}}^{\epsilon_i} f(x|H_y)dx, \text{ for } i = 0, 1, \cdots, l-2. \tag{16}$$

where $f(x|H_0)$ and $f(x|H_1)$ denote the pdf's of T under H_0 and H_1 respectively; let $\epsilon_{-1} = -\infty$ and $\epsilon_{l-1} = +\infty$, hence $P(\epsilon_{-1} \leq T < \epsilon_0) = P(T < \epsilon_0)$, $P(\epsilon_{l-2} \leq T < \epsilon_{l-1}) = P(T \geq \epsilon_{l-2})$.

The Section III-A shows the case that $l = 2$. Next, we derive the cooperative false alarm probability and cooperative detection probability when $l = 3$. According to quantization theory, the two quantized thresholds separate the whole range of T into three regions: Region 0, Region 1 and Region 2. Correspondingly, there are three quantization levels which are denoted by q_0, q_1, and q_2 respectively. In addition, q_0, q_1, and q_2 can be normalized to $\beta_0 = 0$, $\beta_1 = (q_1 - q_0)/(q_2 - q_0)$, and $\beta_2 = 1$ respectively. For convenience, β_0, β_1, and β_2 are directly used in following discussion instead of q_0, q_1, and q_2. When compared with decision fusion rule, the value range of N is $[0, K]$. For the tri-nomial distribution, the related probability mass functions are

$$\begin{aligned} &P(k_0\beta_0 + k_1\beta_1 + k_2\beta_2 = N|H_y) = P(k_1 + k_2\beta_2 = N|H_y) \\ &= \frac{K!}{k_0!k_1!k_2!} P(T < \epsilon_0|H_y)^{k_0} P(\epsilon_0 \leq T < \epsilon_1|H_y)^{k_1} P(T \geq \epsilon_1|H_y)^{k_2} \end{aligned} \tag{17}$$

where k_0, k_1, and k_2 denote the number of the value of T falling in Region 0, Region 1, and Region 2 respectively, and $k_0 + k_1 + k_2 = K$.

Next, we show the conditions that ensure $k_0\beta_0 + k_1\beta_1 + k_2\beta_2 \geq N$ holding for any given value of N. $k_0\beta_0 + k_1\beta_1 + k_2\beta_2 \geq N$ is equivalent to $k_1\beta_1 + k_2 \geq N$. We can derive that when $k_2 \geq \max(0, \lceil \frac{N-K\beta_1}{1-\beta_1} \rceil)$ and $k_1 \geq \max(0, \lceil \frac{N-k_2}{\beta_1} \rceil)$,[2] $k_1\beta_1 + k_2 \geq N$. Accordingly, the related cumulative distribution functions are

$$P(k_0\beta_0 + k_1\beta_1 + k_2\beta_2 \geq N|H_y) = \sum_{k_2=a_2}^{K} P(T \geq \epsilon_1|H_y)^{k_2} \sum_{k_1=a_1}^{K-k_2} \frac{K! P(\epsilon_0 \leq T < \epsilon_1|H_y)^{k_1} P(T < \epsilon_0|H_y)^{k_0}}{k_0! k_1! k_2!} \tag{18}$$

where $a_2 = \max(0, \lceil \frac{N-K\beta_1}{1-\beta_1} \rceil)$, $a_1 = \max(0, \lceil \frac{N-k_2}{\beta_1} \rceil)$. (18) is a closed form function for cooperative false alarm probability and cooperative detection probability under tri-level quantization. Sequentially, we present the cooperative false alarm probability and cooperative detection probability under l-level quantization.

Proposition 1: When $l \geq 2$, $q_0 \leq q_1 \leq q_2 \leq \cdots \leq q_{l-1}$, $\int_{\epsilon}^{+\infty} f(x|H_0)dx < \int_{\epsilon}^{+\infty} f(x|H_1)dx$ for $\forall \epsilon \in (-\infty, +\infty)$, and $\forall N \in [0, K]$, the cooperative false alarm probability and cooperative detection probability of l-level quantization are given by

$$\begin{aligned} P_Y(N) &= P(k_0\beta_0 + k_1\beta_1 + \cdots + k_{l-1}\beta_{l-1} \geq N|H_y) \\ &= \sum_{k_{l-1}=a_{l-1}}^{K} P(T \geq \epsilon_{l-1}|H_y)^{k_{l-1}} \cdots \sum_{k_2=a_2}^{K-\sum_{i=3}^{l-1} k_i} P(\epsilon_1 \leq T < \epsilon_2|H_y)^{k_2} \\ &\quad \times \sum_{k_1=a_1}^{K-\sum_{i=2}^{l-1} k_i} \frac{K!}{k_0! k_1! k_2! \cdots k_{l-1}!} P(\epsilon_0 \leq T < \epsilon_1|H_0)^{k_1} P(T < \epsilon_0|H_y)^{k_0}, \end{aligned} \tag{19}$$

where

$$\beta_i = \frac{q_i - q_0}{q_{l-1} - q_0}, \tag{20}$$

$$a_j = \begin{cases} \max\left(0, \lceil (N - K\beta_{j-1})/(\beta_j - \beta_{j-1}) \rceil\right), j = l-1. \\ \max\left(0, \left\lceil \left[N - \sum_{i=j+1}^{l-1} k_i\beta_i - (K - \sum_{i=j+1}^{l-1} k_i)\beta_{j-1}\right] / (\beta_j - \beta_{j-1}) \right\rceil\right), 1 \leq j \leq l-2. \end{cases} \tag{21}$$

Proof: (19) can be obtained by extending the derivation of functions (15)(18) under $l = 2, 3$ to the case $l > 3$. ■

[2] $\lceil \frac{N-K\beta_1}{\beta_2-\beta_1} \rceil = \lceil \frac{N-K\beta_1}{1-\beta_1} \rceil$, $\lceil \frac{N-k_2\beta_2}{\beta_1} \rceil = \lceil \frac{N-k_2}{\beta_1} \rceil$.

Mathematically, for N, let the set $\mathcal{R} := \{\sum_{i=0}^{l-1} k_i\beta_i : \forall\{k_0, k_1, \cdots, k_{l-1}\}, \sum_{i=0}^{l-1} k_i = K\}$ and the maximum number of elements in $\mathcal{R}$ is $\binom{l+K-1}{K}$. Let $\mathbf{R}$ denote a matrix including every element in $\mathcal{R}$ in ascending order. From (19)-(21), the following *Proposition 2* is derived directly.

Proposition 2: When $N \in [0, K]$, (19) is the non-increasing function of N; when $N \in \mathcal{R}$, (19) is the decreasing function of N.

According to *Proposition 2*, the cooperative false alarm probability and cooperative detection probability can be controlled by adjusting N in (19), rather than enumerating the permutation of $\{k_0, k_1, \cdots, k_{l-1}\}$ in (5) for (6) (7). Thus, (19) suits for NP criterion and CDR criterion feasibly.

C. Study of Quantization based Sensing Performance

This subsection explains the relation between the combined test statistics and the probability mass in detail. And then based on our analysis, we show the computation of sensing performance is simplified in Uniform quantization scheme. Let $\mathbf{K}$ be the $b \times l$ matrix where $b = \binom{l+K-1}{K}$; $\mathbf{K}(m, n)$ = $k_{m,n}$ is the element at m-th row and n-th column. Given m, $k_{m,1}, k_{m,2}, \cdots, k_{m,l}$ means a permutation of $\{k_0, k_1, \cdots, k_{l-1}\}$ where $\sum_{n=0}^{l-1} k_n = K$. Namely, the rows of $\mathbf{K}$ represent every realization of user's reports. Since $\mathbf{K}$ can be obtained effortlessly, consider $\mathbf{K}$ is available. Then each realization of combination of quantized test statistics is given by

$$\mathbf{Q}^{+} = \mathbf{K}\mathbf{q} \tag{22}$$

where $\mathbf{q} = [q_0, q_2, \cdots, q_{l-1}]^T$. Actually, $\forall m$, $\mathbf{Q}^{+}(m) \in \mathcal{R}$. Correspondingly, we have

$$\mathbf{P}_Y^{+}(m) = K! \prod_{n=1}^{l} \frac{\mathbf{p}_y(n)^{\mathbf{K}(m,n)}}{\mathbf{K}(m, n)!}. \tag{23}$$

which is interpreted as the probability mass related to $\mathbf{Q}^{+}(m)$. When $\mathbf{Q}$ means the matrix that the elements of $\mathbf{Q}^{+}$ in ascending order, it exists

$$\mathbf{P}_\pi \mathbf{Q}^{+} = \mathbf{Q} \tag{24}$$

where $\mathbf{P}_\pi$ is the permutation matrix and it is available. Then, we can obtain

$$\mathbf{P}_\pi \mathbf{P}_Y^{+} = \mathbf{P}_Y. \tag{25}$$

Note, for two arbitrary realizations of $\{k_0, k_1, \cdots, k_{l-1}\}$, it is possible that their combined quantized test statistics have the same value. In other words, $\mathbf{Q}$ may contain elements which

have the identical value. For the elements with the same value N in $\mathbf{Q}$, the corresponding index is between $\min(\arg\min_n |\mathbf{Q}(n) - N|)$ and $\max(\arg\min_n |\mathbf{Q}(n) - N|)$. Thus, when $N \in \mathcal{R}$, the probability mass function for CSS is

$$P(k_0\beta_0 + k_1\beta_1 + \cdots + k_{l-1}\beta_{l-1} = N|H_y) = \sum_{n=\min(\arg\min_n |\mathbf{Q}(n)-N|)}^{\max(\arg\min_n |\mathbf{Q}(n)-N|)} \mathbf{P}_Y(n). \tag{26}$$

Consequently, we have the following corollary.

Corollary 1: When $l \geq 2$, $q_0 \leq q_1 \leq q_2 \leq \cdots \leq q_{l-1}$, $\int_\epsilon^{+\infty} f(x|H_0)dx < \int_\epsilon^{+\infty} f(x|H_1)dx$ for $\forall\epsilon \in (-\infty, +\infty)$, and $\forall N \in [0, K]$, the cooperative false alarm probability and cooperative detection probability are given by

$$P_Y(N) = \sum_{n=b_0}^{b} \mathbf{P}_Y(n) \tag{27}$$

where $\mathbf{P}_Y(n)$ is given by (25), $b = \binom{l+K-1}{K}$, and

$$b_0 = \min(\arg\max_n \frac{1}{\mathbf{Q}(n) - N}). \tag{28}$$

Proof: According to (22)-(26), *Corollary 1* is derived evidently. ■

Theoretically, (26) is the probability mass function in terms of the combined test statistics $(k_0\beta_0 + k_1\beta_1 + \cdots + k_{l-1}\beta_{l-1})$. From (26) (27), it is clear to see that the value of quantization levels affects the quantity and the order of elements in $\mathbf{Q}$, decides the probability mass function in turn (26), and determines the final cumulative distribution function (27). Accordingly, when the Uniform quantization scheme is employed, the expression and calculation complexity for cooperative false alarm probability and cooperative detection probability are simplified and reduced respectively. For uniform quantization scheme, the following function is presented for performance evaluation.

$$\mathbf{P}_Y = \underbrace{\mathbf{p}_y * \mathbf{p}_y * \cdots * \mathbf{p}_y}_{K} \tag{29}$$

where $*$ denotes the convolution operation; $\mathbf{P}_Y$ is matrix of c-by-1 where $c = K(l-1) + 1$. Similarly as the derivation of *Corollary 1*, *Corollary 2* is obtained.

Corollary 2: When $l \geq 2$, $q_0 \leq q_1 \leq q_2 \leq \cdots \leq q_{l-1}$, $\int_\epsilon^{+\infty} f(x|H_0)dx < \int_\epsilon^{+\infty} f(x|H_1)dx$ for $\forall\epsilon \in (-\infty, +\infty)$, and $\forall N \in [0, K]$, the cooperative false alarm probability and cooperative

detection probability are given by

$$P_Y(N) = \sum_{n=c_0}^{c} \mathbf{P}_Y(n) \tag{30}$$

where $\mathbf{P}_Y$ is given by (29) respectively, and

$$c_0 = \arg\max_n \frac{1}{\mathbf{R}(n) - N}. \tag{31}$$

Proof: The convolution operation in (29) guarantees that the probability which is the element in $\mathbf{P}_Y$ one-on-one correspond to every realization of $(k_0\beta_0 + k_1\beta_1 + \cdots + k_{l-1}\beta_{l-1})$ which is in ascending order. Hence, we can arrive at *Corollary 2*. ■

Additionally, $\mathbf{P}_Y$ can be obtained by employing FFT transform. Let $\boldsymbol{\mathcal{P}}_0$ and $\boldsymbol{\mathcal{P}}_1$ be the matrices of size $C \times 1$ where $C = 2\lceil c/2 \rceil$. $\forall i \in \{1, 2, \cdots, K\}$, $\boldsymbol{\mathcal{P}}_0 = [\mathbf{p}_0^T, \mathbf{0}^T]^T$ and $\boldsymbol{\mathcal{P}}_1 = [\mathbf{p}_1^T, \mathbf{0}^T]^T$.

$$\begin{aligned} \mathbf{P}_Y &= IDFT(diag(DFT(\boldsymbol{\mathcal{P}}_y))^K \mathbf{1}_{C\times 1}) = \mathbf{W}^{-1}(diag(\mathbf{W}\boldsymbol{\mathcal{P}}_y)^K \mathbf{1}_{C\times 1})) \\ &= \mathbf{W}^{-1} diag(\mathbf{W}\boldsymbol{\mathcal{P}}_y)^K \mathbf{1}_{C\times 1} \end{aligned} \tag{32}$$

where $DFT(\mathbf{x})$ and $IDFT(\mathbf{x})$ denote the discrete Fourier transform (DFT) of $\mathbf{x}$ and inverse discrete fourier transform of $\mathbf{x}$ respectively; $diag(\mathbf{x})$ denotes for a diagonal matrix whose diagonal entries are same as the elements of vector $\mathbf{x}$ in turn; and $\mathbf{W}$ is the DFT matrix. Therefore, (32) provides an efficient method to obtain $\mathbf{P}_Y$.

D. Normal Approximation

This subsection presents the Normal approximation of cooperative false alarm probability and cooperative detection probability. The Normal approximation provides an efficient approach to evaluate the performance of multi-level quantization based CSS under given $\{\epsilon_0, \epsilon_1, \cdots, \epsilon_{l-2}\}$ and $\{q_0, q_1, \cdots, q_{l-1}\}$. Additionally, the Normal approximation is used for performance optimization in next section. Let $\mathbf{p}_y = [P(T < \epsilon_0|H_y), P(\epsilon_0 \le T < \epsilon_1|H_y), \cdots, P(T \ge \epsilon_{l-2}|H_y)]^T$, $\boldsymbol{\beta} = [\beta_0, \beta_2, \cdots, \beta_{l-1}]^T$, and $\mathbf{P}_y = \mathbf{1}_{K\times 1}\mathbf{p}_y^T$. For $l \ge 2$, we have mean μ_0 and standard deviation σ_0 for state H_0, and mean μ_1 and standard deviation σ_1 for state H_1,

$$\mu_y = \mathbf{p}_y^T \boldsymbol{\beta}, \tag{33}$$

$$\sigma_y = \sqrt{\frac{(\boldsymbol{\beta} - \mathbf{P}_y\boldsymbol{\beta})^T(\boldsymbol{\beta} - \mathbf{P}_y\boldsymbol{\beta})}{K}} = \sqrt{\frac{\boldsymbol{\beta}^T(\mathbf{I} - \mathbf{P}_y)^T(\mathbf{I} - \mathbf{P}_y)\boldsymbol{\beta}}{K}}. \tag{34}$$

According to Central Limit Theorem (CLT), $\mathbb{T}$ denoting the combination of quantized test statistics follows that,

$$\mathbb{T} \sim \begin{cases} \mathcal{N}(\mu_0, \sigma_0), & H_0 \\ \\ \mathcal{N}(\mu_1, \sigma_1), & H_1 \end{cases} \tag{35}$$

where $\mathcal{N}$ denotes Normal distribution.

Considering the continuity correction, we have to refine the approximation by accounting for the fact that the multinomial distribution is discrete while the Normal distribution is continuous. At first, we take bi-level quantization as an illustration, and then extend to multi-level quantization case. As previously explained, the Binomial distribution is treated as the outcomes from bi-level quantization. According to the previous analysis, the effective value of N in (15) is positive integer, consisting of β_0 and β_1. Thus, regarding the continuity correction, we have

$$\begin{aligned} P(\mathbb{T} > \lfloor N/\beta_1 \rfloor / K | H_y) = P(\mathbb{T} \geq \lceil N/1 \rceil / K | H_y) &= P(\mathbb{T} \geq (\lfloor N/1 \rfloor + 1)/K | H_y) \\ &\approx P(\mathbb{T}' \geq (\lfloor N \rfloor + 0.5)/K | H_y) = P(\mathbb{T}' \geq (\lceil N \rceil - 0.5)/K | H_y) \\ &= \int_{\frac{\lceil N \rceil}{K} - \frac{0.5}{K}}^{+\infty} f_{\mathcal{N}}(x|H_y) dx \end{aligned} \tag{36}$$

where $\mathbb{T}' \in [0,1]$; $f_{\mathcal{N}}(.)$ denotes the pdf of Normal distribution. Obviously, when $K \to +\infty$, the value of $(\lceil N \rceil / K - 0.5/K)$ can be considered as the continuous positive number for evaluating the cooperative false alarm probability and cooperative detection probability. Consequently, we arrive at the following Normal approximation for $l = 2$,

$$P_Y(N) \approx Q\left(\frac{\lceil N \rceil / K - 0.5/K - P(T \geq \epsilon_0 | H_y)}{\sqrt{P(T \geq \epsilon_0 | H_y) P(T < \epsilon_0 | H_y)/K}} \right) \tag{37}$$

where $0.5/K$ is the continuity correction; $Q(.)$ means the Q-function. Sequentially, the following corollary is derived,

Corollary 3: Given $l \geq 2$ and $q_0 \leq q_1 \leq q_2 \leq \cdots \leq q_{l-1}$, for $\forall N \in [0, K]$, the Normal approximations of cooperative false alarm probability and cooperative detection probability are given by

$$P_Y(N) \approx Q\left(\frac{2K\mathbb{N} - 2K\mathbf{p_y}^T \boldsymbol{\beta} - \beta_1}{2\sqrt{\boldsymbol{\beta}^T (\mathbf{I} - \mathbf{P_y})^T (\mathbf{I} - \mathbf{P_y}) \boldsymbol{\beta} K}} \right), \tag{38}$$

where

$$\mathbb{N} = \arg \max_n \frac{1}{\mathbf{R}(n)/K - N}. \tag{39}$$

Proof: At the fusion center, the value of realization of $\mathbb{T}$ belongs to set $\mathcal{C} := \{x/K : \forall x \in \mathcal{R}\}$. Based on (39), it is observed that $P(\mathbf{R}(n) - \mathbf{R}(n-1) = \beta_1 - \beta_0)$ has the highest probability for $\forall n > 1$. Thus, similarly as the continuity correction $0.5/K$ in bi-level quantization case, it is regarded that the continuity correction denoted by Δ is as follows for multi-level quantization case,

$$\Delta = \frac{\beta_1 - \beta_0}{2K} = \frac{\beta_1}{2K}. \tag{40}$$

Based on (33)-(37)(40), we arrive at *Corollary 3*. ■

IV. Quantization and Optimization

The derived *Proposition 1* provides a feasible bridge between the cooperative sensing performance and the quantization configurations. On the one hand, (19) can correctly evaluate the performance of CSS, when the quantization levels and quantized thresholds are arbitrary values under $q_0 \leq q_1 \leq q_2 \leq \cdots \leq q_{l-1}$; on the other hand, the quantization levels and quantized thresholds facilitate the understanding of the determination of CSS performance rather than likelihood ratio. Hence, we can utilize (19) to reasonably optimize the related quantization configuration for high sensing performance. For example, in bi-level quantization based CSS, (14) can correctly evaluate the performance of CSS for a certain value of sensing threshold, but there exists an optimal sensing threshold which determines the quantization levels for achieving the optimal sensing performance [18].

A. Minimum Mean Square Error (MMSE) based Quantization Scheme and Optimization

As a merits of adopting quantization theory to evaluate the CSS, there have been a large number of literatures on quantization for direct use. According to [19], the Lloyd's quantization scheme can achieve the minimum information loss of quantization. When the given pdf of test statistics is $g(x)$, the quantization levels $\{q_0, q_1, \cdots, q_{l-1}\}$ are given by (41) to achieve the MMSE caused by quantization.

$$q_j = \frac{\int_{\epsilon_{j-1}}^{\epsilon_j} x g(x) dx}{\int_{\epsilon_{j-1}}^{\epsilon_j} g(x) dx} \text{ for } j = 0, 1, \cdots, l-1. \tag{41}$$

The mean square error (MSE) under quantization is written by,

$$M = \sum_{j=0}^{l-1} \int_{\epsilon_{j-1}}^{\epsilon_j} (x - q_j)^2 g(x) dx. \tag{42}$$

Substituting (41) into (42) yields

$$M = \sum_{j=0}^{l-1} \int_{\epsilon_{j-1}}^{\epsilon_j} \left(x - \frac{\int_{\epsilon_{j-1}}^{\epsilon_j} x g(x) dx}{\int_{\epsilon_{j-1}}^{\epsilon_j} g(x) dx} \right)^2 g(x) dx. \tag{43}$$

Then, we arrive at the following programming,

$$\min_{\epsilon_0, \epsilon_1, \cdots, \epsilon_{l-2}} \quad M(\epsilon_0, \epsilon_1, \cdots, \epsilon_{l-2}) = \sum_{j=0}^{l-1} \int_{\epsilon_{j-1}}^{\epsilon_j} \left(x - \frac{\int_{\epsilon_{j-1}}^{\epsilon_j} x g(x) dx}{\int_{\epsilon_{j-1}}^{\epsilon_j} g(x) dx} \right)^2 g(x) dx \quad . \tag{44}$$

In CR, there are two pdf's $f(x|H_0)$ and $f(x|H_1)$ for test statistics T. Hence, $g(x)$ is given by

$$g(x) = \frac{f(x|H_0) + w f(x|H_1)}{1 + w} \tag{45}$$

where w denotes the cost or weight. The involved cooperative SUs have to quantize the test statistics T under H_0 and H_1 concurrently so that the fusion center can distinguish the real state of primary user with high correct probability. The corresponding visual interpretation is that: when treating the outline of the outputs of $f(x|H_0)$ and $f(x|H_1)$ in terms of x as the waveform of target signal, the objective of quantization is to make the fusion center can recover this waveform with minimum distortion. Thus, consider $w = 1$ in following discussion. (41) (45) show a feasible solution to obtain the quantization levels under certain quantization thresholds $\{\epsilon_0, \epsilon_1, \cdots, \epsilon_{l-2}\}$.

[19] shows the existence and uniqueness of the optimal quantization thresholds $\{\epsilon_0, \epsilon_1, \cdots, \epsilon_{l-2}\}$ from which the optimal quantization levels $\{q_0, q_1, \cdots, q_{l-1}\}$ can be obtained directly. Thus, we can obtain the optimal sensing configuration including the optimal quantization levels and the optimal sensing thresholds. When comparing the exclusive search employed for the optimal quantization configuration of Lloyd's scheme in current studies , we show a method to obtain the optimum thresholds and quantization levels efficiently. According to [19], when the quantization thresholds are optimal, there exists

$$\epsilon_j = (q_j + q_{j+1})/2, \text{for } \forall j \in \{0, 1, \cdots, l-2\}. \tag{46}$$

From (41) (46), we have

$$\int_{\frac{q_{j-2}+q_{j-1}}{2}}^{\frac{q_{j-1}+q_j}{2}} (x - q_{j-1}) g(x) dx = 0, \text{for } \forall j \in \{2, 3, \cdots, l-2\}. \tag{47}$$

which means, given q_{j-2} and q_{j-1}, the optimal q_j makes equation (47) hold.

Evidently, $\int_{(q_{j-2}+q_{j-1})/2}^{(q_{j-1}+q_j)/2}(x-q_{j-1})g(x)dx$ is a monotone increasing function of q_j. Hence, $\phi_j(q_j) = \int_{(q_{j-2}+q_{j-1})/2}^{(q_{j-1}+q_j)/2}(x-q_{j-1})g(x)dx$ is quasi-convex function of q_j and has the minimal value when $\phi_j(q_j) \geq 0$. From (41)(46)(47), we have

$$q_0^+(\epsilon_0) = \int_{-\infty}^{\epsilon_0} xg(x)dx / \int_{\infty}^{\epsilon_0} g(x)dx, \tag{48}$$

$$q_1^+(\epsilon_0) = 2\epsilon_0 - q_0^+(\epsilon_0), \tag{49}$$

$$\begin{aligned} q_2^+(\epsilon_0) &= \arg\min_{q_2} \ \phi_2(q_2, q_1^+(\epsilon_0), q_0^+(\epsilon_0)) \\ st.\ q_1^+ &\leq q_2 < +\infty, \phi_2(q_2, q_1^+(\epsilon_0), q_0^+(\epsilon_0)) \geq 0. \end{aligned} \tag{50}$$

Let $\{q_0^+, q_1^+, \cdots, q_{l-1}^+\}$ denote the calculated quantization levels under a given value of ϵ_0. In turn, we can obtain q_{l-1}^+ given by

$$\begin{aligned} q_{l-1}^+(\epsilon_0) &= \arg\min_{q_{l-1}} \ \phi_{l-1}(q_{l-1}, q_{l-2}^+(\epsilon_0), q_{l-3}^+(\epsilon_0)) \\ st.\ q_{l-2}^+ &\leq q_{l-1} < +\infty, \phi_{l-1}(q_{l-1}, q_{l-2}^+(\epsilon_0), q_{l-3}^+(\epsilon_0)) \geq 0. \end{aligned} \tag{51}$$

Proposition 3: Under $\phi_{l-1} \geq 0$, two propositions are given: *i*) when ϵ_0 is variable, the value of the l-th quantization level monotonically increases as ϵ_0 increasing; *ii*) when q_0 is variable, the value of the l-th quantization level monotonically increases as q_0 increasing.

Proof: From (48)-(51), it is obvious that q_{l-1}^+ monotonically increases in term of the variable ϵ_0 is increasing, q_{l-1}^+ monotonically increases in term of the variable q_0 is increasing. ■

In addition, when $\min_{q_{l-1}} \phi_{l-1} = 0$, ϵ_0 achieves the optimal value. Summarily, for $\forall \epsilon_0$, obtain $\{q_0^+(\epsilon_0), q_1^+(\epsilon_0), \cdots, q_{l-1}^+(\epsilon_0)\}$; when $\phi_{l-1}(q_{l-1}^+(\epsilon_0), q_{l-2}^+(\epsilon_0), q_{l-3}^+(\epsilon_0)) = 0$, $\{q_0^+(\epsilon_0), q_1^+(\epsilon_0), \cdots, q_{l-1}^+(\epsilon_0)\}$ are optimal. Thus, the original optimization problem with $(2l-1)$ variables has been transformed to an optimization problem with a variable ϵ_0. Consequently, by employing the bisection method for quasiconvex optimization [21], we can obtain the optimal ϵ_0 at first. Sequentiality, by using the optimal ϵ_0, the other optimal sensing thresholds and the corresponding optimal quantization levels can be obtained directly.

B. Optimization based on Normal Approximation

Under given quantized thresholds, the following method is proposed to obtain the optimal quantization levels by using the results of Normal approximation in Section III-D. From (38), we have

$$P_d = Q\left(f_o\right) \tag{52}$$

where

$$f_o = \frac{Q^{-1}(P_f)\sigma_0 + \mu_0 - \mu_1}{\sigma_1}. \tag{53}$$

The smaller value of f_o gains the higher detection probability. Letting $\mathbf{q} = [q_0, q_2, \cdots, q_{l-1}]^T$ replace $\boldsymbol{\beta}$ in (33)-(34) yields

$$\mu_y = \mathbf{p}_y^T \mathbf{q}, \tag{54}$$

$$\sigma_y = \sqrt{\frac{\mathbf{q}^T(\mathbf{I} - \mathbf{P}_y)^T(\mathbf{I} - \mathbf{P}_y)\mathbf{q}}{K}}. \tag{55}$$

Substituting (54)(55) into (53) yields

$$f_o(\mathbf{q}) = \frac{Q^{-1}(P_f)\sigma_0 - (\mu_1 - \mu_0)}{\sigma_1} = \frac{Q^{-1}(P_f)\sqrt{\mathbf{q}^T(\mathbf{I} - \mathbf{P_0})^T(\mathbf{I} - \mathbf{P_0})\mathbf{q}/K} - \mathbf{q}^T(\mathbf{p_1} - \mathbf{p_0})}{\sqrt{\mathbf{q}^T(\mathbf{I} - \mathbf{P_1})^T(\mathbf{I} - \mathbf{P_1})\mathbf{q}/K}}. \tag{56}$$

Thus, the following programming is obtained,

$$\min_{\mathbf{q}} \; f_o(\mathbf{q}) \; st. \; \mathbf{q} \succeq \mathbf{0} \tag{57}$$

which can be efficiently solved by using the method introduced in [16]. Consequently, given sensing thresholds, the optimal quantization levels can be obtained from our programming efficiently.

V. Examples

For clarity of expression, let LO denote the results of CSS in Lloyd's quantization scheme with the proposed optimization method; NA means the results of CSS from Normal approximation; AF represents the sensing results when T is analog forwarded to fusion center without quantization; LRT expresses the results under LRT test and the quantized thresholds are same as the LO's; and let NO denote the results of Normal-Approximation-based-optimization at the case that the quantized thresholds are same as the LO's.

A. Normal Data

Let T follow the Normal distribution under both states H_0 and H_1,

$$f(x|H_0) = \frac{1}{\sqrt{2\pi}\sigma_{T,0}} \exp(-\frac{(x - \mu_{T,0})^2}{\sigma_{T,0}^2}), \tag{58}$$

$$f(x|H_1) = \frac{1}{\sqrt{2\pi}\sigma_{T,1}} \exp(-\frac{(x-\mu_{T,1})^2}{\sigma_{T,1}^2}) \tag{59}$$

where $\mu_{T,0}$, $\mu_{T,1}$, $\sigma_{T,0}$, and $\sigma_{T,1}$ are the mean and standard deviation under H_0 and H_1 respectively. Fig. 2-Fig. 7 show the results of cooperative sensing under quantization with $\mu_{T,0} = 0$, $\mu_{T,1} = 1$, $\sigma_{T,0} = 1.5$, and $\sigma_{T,1} = 2$.[3] It is apparent to obtain the following conclusions: *i*) the LO case can achieve the high sensing performance when compared with the AF case; *ii*), according to Fig. 2, Fig. 4, and Fig. 6, the Normal approximation provides good result for analyzing the CSS's performance under quantization. And the results of Normal approximation become more accurate as the quantity of quantization levels or the number of cooperative users increasing; *iii*) the NO case can achieve comparable sensing results to LRT case, according to Fig. 3, Fig. 5, and Fig. 7.

B. Rayleigh Data

Consider T follows Normal distribution under H_0, and Rayleigh distribution under H_1. The related pdf under H_1 is given by

$$f(x|H_1) = \frac{1}{\sigma_{T,1}^2} \exp(-\frac{x^2}{2\sigma_{T,1}^2}), x \geq 0\cdot \tag{60}$$

Fig. 8-Fig. 11 indicate the results of CSS under quantization with $\mu_{T,0} = 1$, $\sigma_{T,0} = 0.5$, and $\sigma_{T,1} = 1$. It is evident that we can obtain the conclusions same as the Normal data case's, even the Rayleigh data is employed. Hence, it proves that our derived methods and functions can suit for different distributions of test statistics.

VI. Conclusion

In this paper, the performance of CSS is studied considering the impact of quantization. We derive the closed form functions to calculate the cooperative false alarm probability and cooperative detection probability, which ensure the performance of CSS with limited quantization levels is no longer intractable, especially, for optimization. Additionally, the related optimization methods are provided to improve the performance of CSS.

[3] In practice, the values of $\mu_{T,0}$ and $\mu_{T,1}$ can be normalized to 0 and 1 respectively. Here, we prefer $\sigma_{T,0} = 1.5$ and $\sigma_{T,1} = 2$ randomly with considering that $\sigma_{T,0} < \sigma_{T,1}$ in CR scenario. For example, the CSS employs energy detector.

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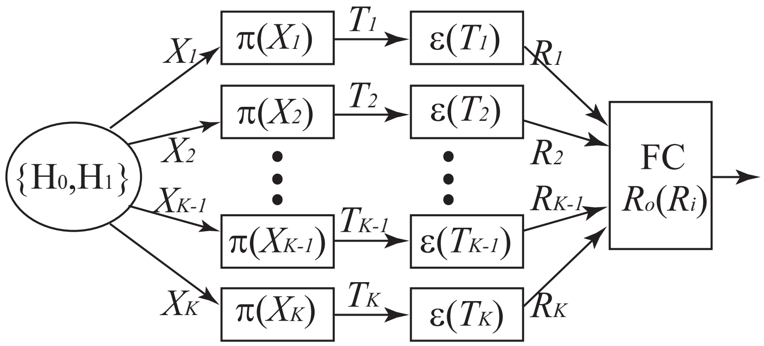

Fig. 1. Illustration of CSS in CR. FC is short for fusion center. $\pi(.)$ denotes the pre-processing operation and $\varepsilon(.)$ represents the quantization operation.

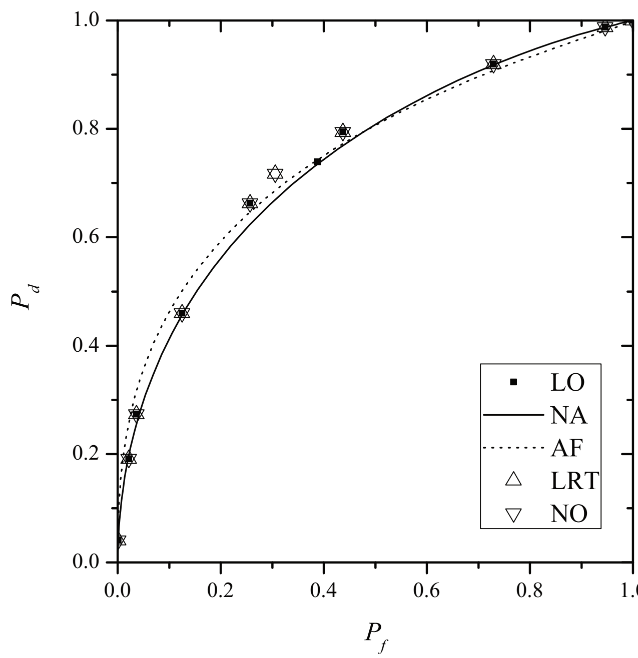

Fig. 2. The ROC line with $K = 3$, $l = 3$. Marks denote the CSS's performance under the different discrete combined test statistics.

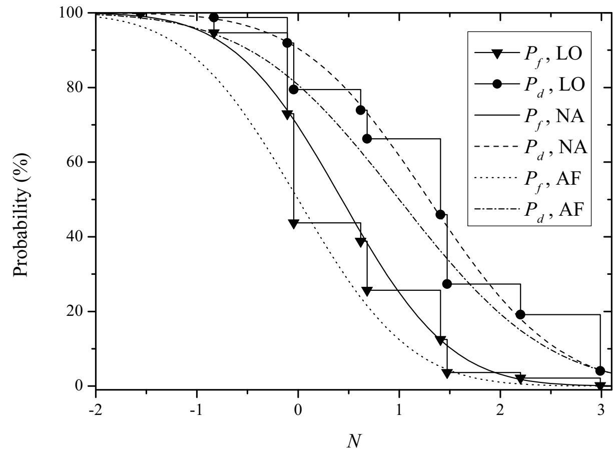

Fig. 3. The sensing performance under different N, with $K = 3$, $l = 3$. Marks denote the CSS's performance under the different discrete combined test statistics.

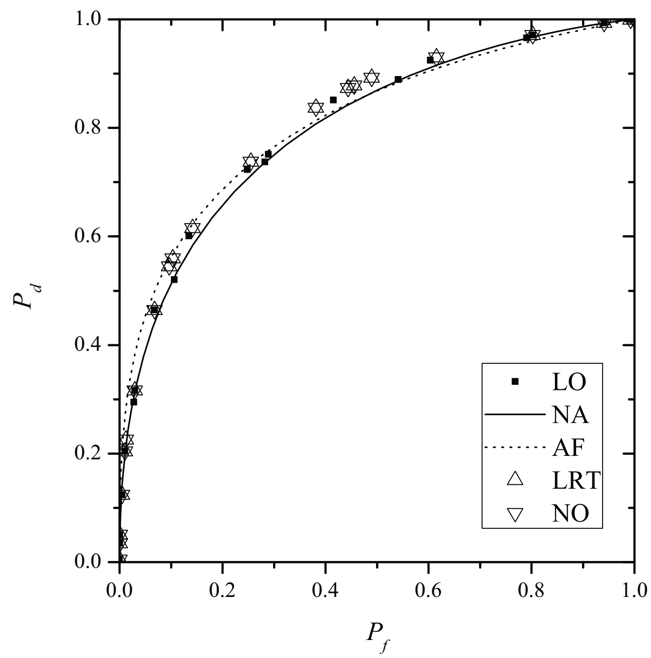

Fig. 4. The ROC line with $K = 5$, $l = 3$. Marks denote the CSS's performance under the different discrete combined test statistics.

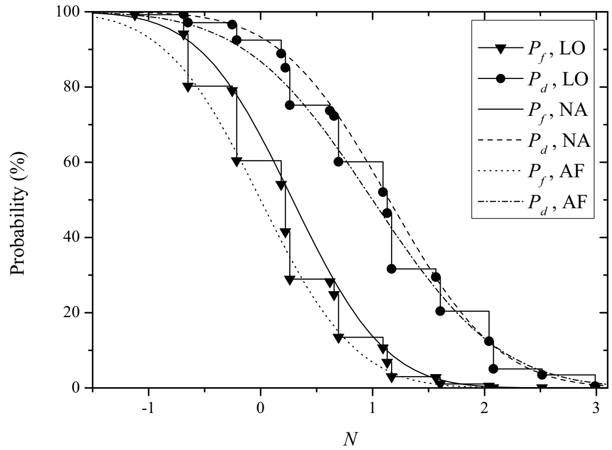

Fig. 5. The sensing performance under different N, with $K = 5$, $l = 3$. Marks denote the CSS's performance under the different discrete combined test statistics.

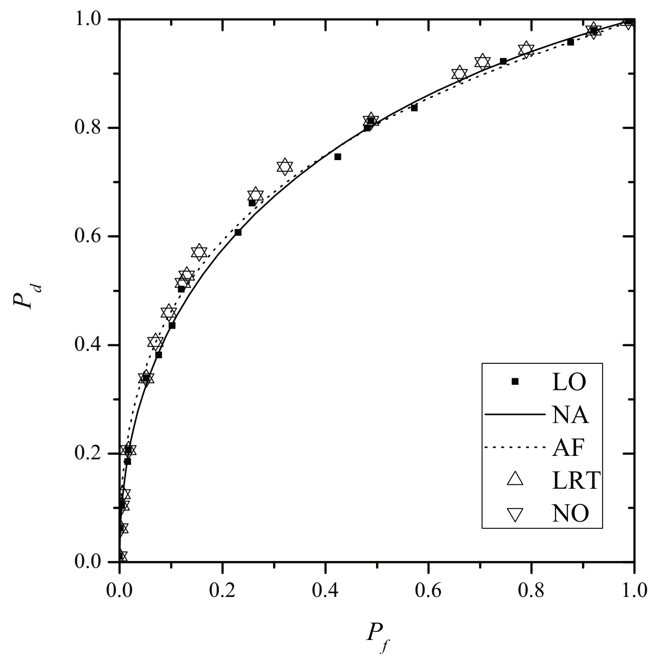

Fig. 6. The ROC line with $K = 3$, $l = 4$. Marks denote the CSS's performance under the different discrete combined test statistics.

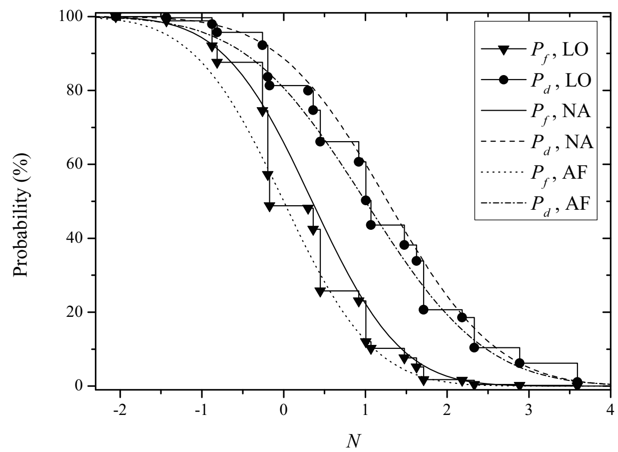

Fig. 7. The sensing performance under different N, with $K = 3$, $l = 4$. Marks denote the CSS's performance under the different discrete combined test statistics.

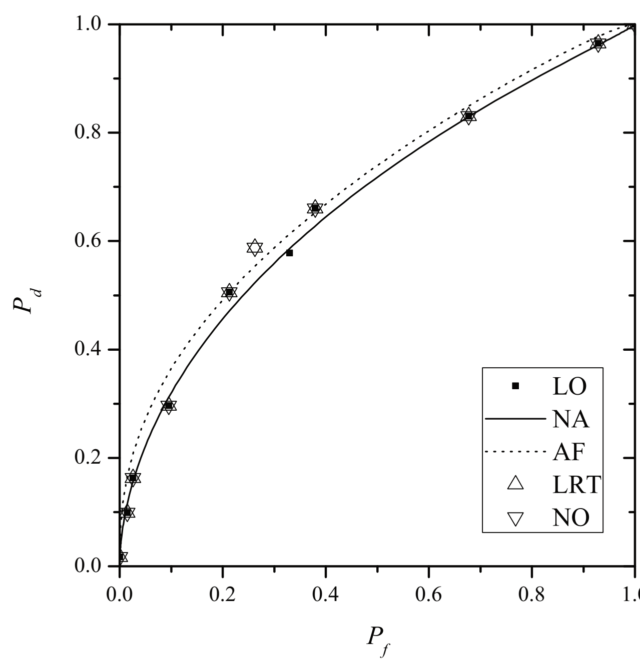

Fig. 8. The ROC line with $K = 3$, $l = 3$. Marks denote the CSS's performance under the different discrete combined test statistics.

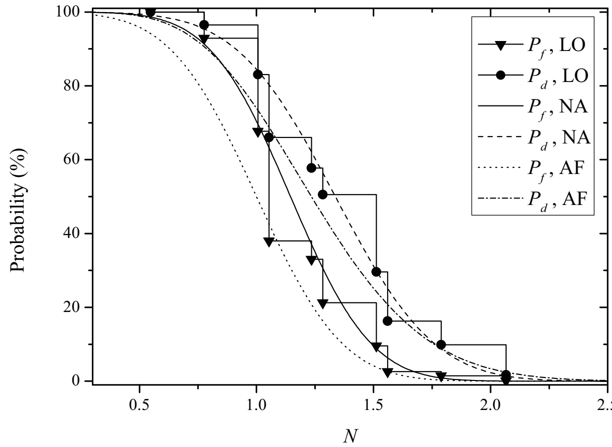

Fig. 9. The sensing performance under different N, with $K = 3$, $l = 3$. Marks denote the CSS's performance under the different discrete combined test statistics.

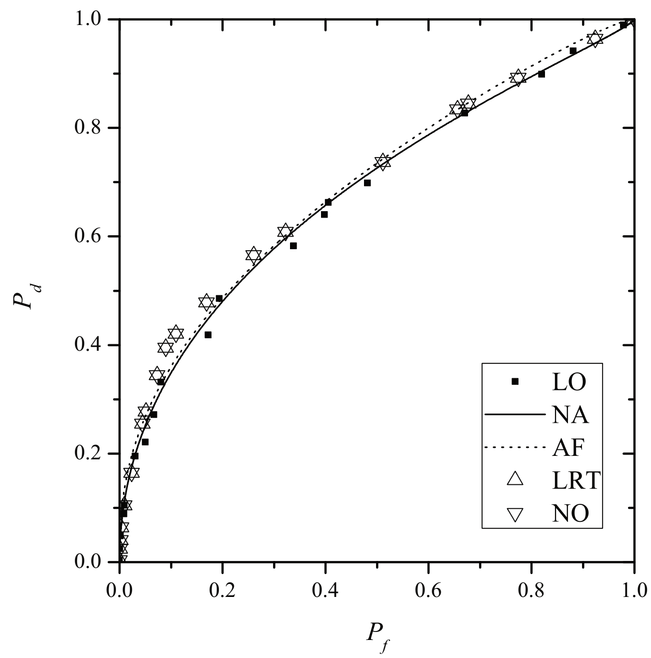

Fig. 10. The ROC line with $K = 3$, $l = 4$. Marks denote the CSS's performance under the different discrete combined test statistics.

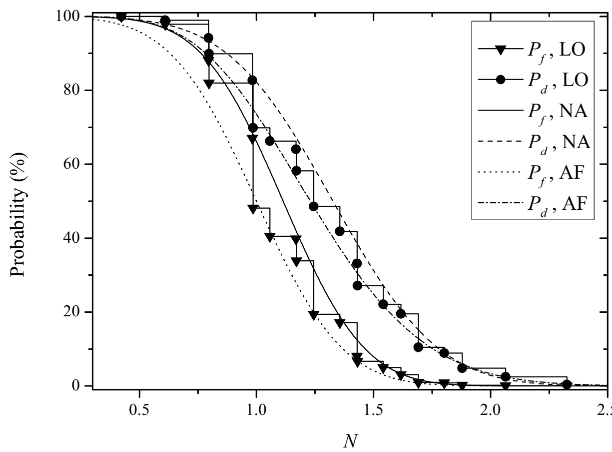

Fig. 11. The sensing performance under different N, with $K = 3$, $l = 4$. Marks denote the CSS's performance under the different discrete combined test statistics.